Title:Monitoring of Delamination Defects - Dynamic Case

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ABSTRACT

The purpose of this paper is to propose an alternative approach to the inverse dynamic analysis problem. Generalizing the so-called VDM (Virtual Distortion Method) approach for dynamic problems, a dynamic influence matrix D concept will be introduced. Pre-computing of the time-dependent matrix D allows for decomposition of the dynamic structural response into components caused by external excitation in undamaged structure (the linear part) and components describing perturbations caused by the internal defects (the non-linear part). As a consequence, analytical formulae for calculation of these perturbations and the corresponding gradients can be derived. The physical meaning of the so-called *virtual distortions* used in this paper can be explained with the help of externally induced strains (non-compatible in general, e.g. caused by piezoelectric transducers, similarly to the effect of non-homogeneous heating). The compatible strains and self-equilibrated stresses are structural responses to these distortions. Assuming possible locations of all potential defects in advance, an optimisation technique with analytically calculated gradients could be applied to solve the problem of the most probable location of defects. The considered damage can affect the local stiffness as well as the mass distribution modification. It is possible to identify the position as well as intensity of several, simultaneously generated defects (delaminations).

ITRODUCTION

The damage detection systems based on array of piezoelectric transducers sending and receiving strain waves are intensively discussed by researchers recently. The signal-processing problem is the crucial point in this concept and a neural network based method is one of the most often suggested approaches to develop a numerically efficient solver for this problem.

An alternative way for these techniques is the VDM approach. The software tool based on this method can be dedicated for different kind of damage, also for delamination defect identification problem, which will be discussed below.

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VDM STATIC APPROACH IN DELAMINATION MONITORING

We shall pose the optimisation problem of structural damage identification (constraining ourselves temporarily to the static case) within the framework of the Virtual Distortion Method (cf. [4]). Let us minimise the following function:

$$\min \sum_{A} \left(\varepsilon_A^M - \varepsilon_A \right)^2 \tag{1}$$

which can be interpreted as an average departure of the total structural strain $\boldsymbol{\varepsilon}_{\mathbf{A}}$ from the in-situ measured strain $\boldsymbol{\varepsilon}_{\mathbf{A}}{}^{\mathbf{M}}$ in damaged locations *A*. Taking advantage of the VDM formulation we can decompose the strain $\boldsymbol{\varepsilon}_{\mathbf{A}}$ into two parts:

$$\varepsilon_A = \varepsilon_A^L + \varepsilon_A^R = \varepsilon_A^L + \sum_{A} D_{Ai} \varepsilon_i^o \tag{2}$$

where $\boldsymbol{\varepsilon}_{\mathbf{A}}{}^{\mathbf{L}}$ denotes the response of undamaged structure, **D** is the influence matrix and $\boldsymbol{\varepsilon}^{\mathbf{o}}$ is the virtual distortion vector. As the component $\boldsymbol{\varepsilon}_{\mathbf{A}}{}^{\mathbf{L}}$ is constant for a given external load, the so-called residual strain component $\boldsymbol{\varepsilon}_{\mathbf{A}}{}^{\mathbf{R}}$ may only be varying in the optimisation process with the virtual distortion $\boldsymbol{\varepsilon}^{\mathbf{o}}$ as the design variable.

We shall measure the structural damage in each member *i* with the help of the coefficient μ_i i.e. with the ratio of cross-sectional areas of a damaged member to the undamaged one. Consequently we have to impose appropriate constraints on this coefficient. As we examine the physical process of deterioration of the member cross-section we are interested in such μ_i, which complies with the following constraints:

$$0 \le \mu_i \le 1 \quad \text{i.e.} \quad 0 \le \frac{\varepsilon_i - \varepsilon_i^o}{\varepsilon_i} \le 1 \tag{3}$$

For delamination problems the coefficient μ will finally (after optimisation) take only two values: 0 (delamination) or 1 (full connection).

The gradients of the objective function and the constraints are expressed in terms of the design variable $\boldsymbol{\varepsilon}^{\mathbf{o}}$ as follows:

$$\nabla f = \frac{\partial f}{\partial \varepsilon_k^o} = -2 \sum_{A} D_{Ak} \left(\varepsilon_A^M - \varepsilon_A \right) \quad \text{and} \quad N = n_{kl} = \frac{\partial g_l}{\partial \varepsilon_k^o} = \frac{\delta_{lk} \varepsilon_l - D_{lk} \varepsilon_l^o}{\left(\varepsilon_l \right)^2} \tag{4}$$

In order to solve the damage identification problem posed by (1) and (3) the Gradient Projection Method (cf. [1], [2], [3]) can be used as optimisation tool. The Gradient Projection Method is based on the idea of projecting the search direction (i.e. the direction in which the objective function value decreases) into the subspace tangent to the active constraints. For the case of linear constraints the optimisation problem can be posed in the following way:

$$\min \ f(x) \tag{5}$$

$$\text{subject to: } g_j(x) = \sum_{i=1}^{n} n_{ji} x_i - b_j \geq 0, \quad j = 1,...,n_g \tag{6}$$

where $n_{kl} = \frac{\partial g_l}{\partial x_k}$ i.e. the gradients of the constraints are stored column-wise. Subscripts i and k run through the number of design variables n whereas subscripts j and l run through the number of constraints n_g.

If we select only the r active constraints then the constraints may be written as follows:

$$g_a = N^T x - b = 0 \tag{7}$$

where the matrix **N** stores gradients of the constraints in columns.

NUMERICAL EXAMPLE – STATIC CASE

In the modelling of contact layer the following conditions have been imposed on the pair of inclined elements (left and right) and on the transversal one in each delaminated cell "*i*" (cf. Fig.1):

- in the case of transversal compression in delaminated cell "*i*":

$$\begin{gathered} \varepsilon_{iR}^{0} = \varepsilon_{iR} \quad \varepsilon_{iL}^{0} = \varepsilon_{iL} \\ \varepsilon_{iN}^{0} = \varepsilon_{iN} \end{gathered} \tag{8}$$

- in the case of non-compressive transversal interactions in delaminated cell "*i*":

$$\begin{gathered} \varepsilon_{iR}^{0} = \varepsilon_{iR} \quad \varepsilon_{iL}^{0} = \varepsilon_{iL} \\ \varepsilon_{iN}^{0} = 0 \end{gathered} \tag{9}$$

where

$$\begin{gathered} \varepsilon_{iR} = \varepsilon_{iR}^{L} + \sum_{j,k} D_{iR,jk} \varepsilon_{jk}^{0} \;\; ; \;\; \varepsilon_{iL} = \varepsilon_{iL}^{L} + \sum_{j,k} D_{iL,jk} \varepsilon_{jk}^{0} \\ \varepsilon_{iN} = \varepsilon_{iN}^{L} + \sum_{j,k} D_{iN,jk} \varepsilon_{jk}^{0} \;\; ; \;\; k = R, L, N \end{gathered} \tag{10}$$

and $D_{iR,jk}$, $D_{iL,jk}$, $D_{iN,jk}$ denote influence vectors describing strains generated in elements: right, left and normal in the cell "*i*" induced due to unit virtual distortion generated in element "*jk*".

The above conditions leads to the following effects modelling the contact problem in the gap generated due to delamination:

- in the case of transversal compression in delaminated cell "*i*" the shear forces vanish in the contact layer,
- in the case of non-compressive transversal interactions in delaminated cell "*i*" the shear forces as well as the transversal forces vanish.

The sets of equations (8), (9) allows determination of virtual distortions modelling shear movement and transversal gap development along the contact layer. The resultant strains in contact elements take the form (10).

The algorithm for the contact problem analysis in delaminated layer is shown in Table 1.

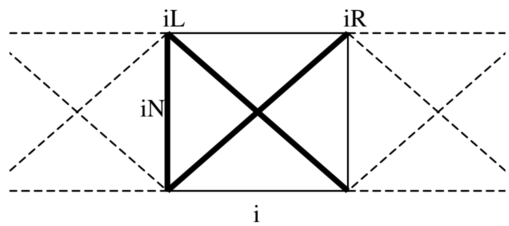


Fig.1 Notation used in description of the contact layer.

Table 1. The algorithm for the contact problem analysis in delaminated layer

INITIALIZATION

Determine:

Linear response undamaged structure ε_{ik}^{L}

influence matrix: $D_{im,jk}$ $\quad m,k = R,L,N$

damage vector $\mu_{iR}, \mu_{iL}, \mu_{iN}$

FOR EACH TIME STEP t:

Assuming transversal compression in delaminated zone equations (8), (10) allow to determine associated virtual distortions (cf.(8)),what leads to the following system of equations Ax=b, where:

$$A = \begin{bmatrix} (1-\mu_{iR})(D_{iR,jR} - \delta_{ij}) & (1-\mu_{iL})D_{iR,jL} \\ (1-\mu_{iR})D_{iL,jR} & (1-\mu_{iL})(D_{iL,jL} - \delta_{ij}) \end{bmatrix} \; ; B = \begin{bmatrix} -(1-\mu_{iR})\varepsilon_{iR}^{L} \\ -(1-\mu_{iL})\varepsilon_{iL}^{L} \end{bmatrix};$$

Update responses for each vertical element in delaminated area:

$$\varepsilon_{iN} = \varepsilon_{iN}^{L} + \sum_{j} D_{iN,jR}\varepsilon_{jR}^{0} + \sum_{j} D_{iN,jL}\varepsilon_{jL}^{0}$$

and check if :

$$\varepsilon_{iN} \geq 0$$

NO

YES

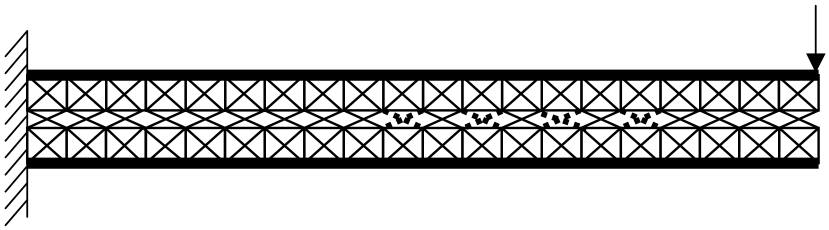
In the case of non-compressive transversal interactions determine associated virtual distortions from equations (9), (10), what leads to the following system of equations Ax=b, where:

$$A = \begin{bmatrix} (1-\mu_{iR})(D_{iR,jR} - \delta_{ij}) & (1-\mu_{iL})D_{iR,jL} & (1-\mu_{iN})D_{iR,jN} \\ (1-\mu_{iR})D_{iL,jR} & (1-\mu_{iL})(D_{iL,jL} - \delta_{ij}) & (1-\mu_{iNi})D_{iL,jN} \\ (1-\mu_{iR})D_{iN,jR} & (1-\mu_{iL})D_{iN,jL} & (1-\mu_{iN})(D_{iN,jN} - \delta_{ij}) \end{bmatrix}; \qquad B = \begin{bmatrix} -(1-\mu_{iR})\varepsilon_{iR}^{L} \\ -(1-\mu_{iL})\varepsilon_{iL}^{L} \\ -(1-\mu_{iN})\varepsilon_{iN}^{L} \end{bmatrix};$$

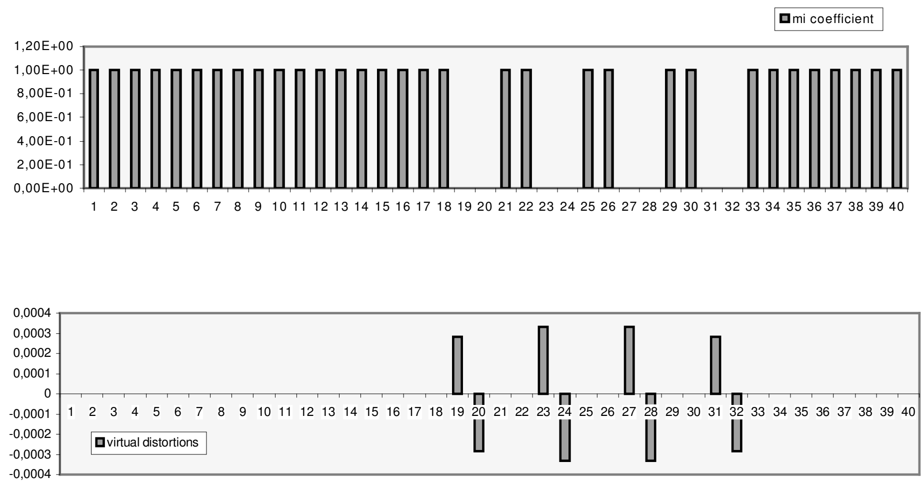
Update responses for each element:

$$\varepsilon_{im} = \varepsilon_{im}^{L} + \sum_{j,k} D_{im,jk}\varepsilon_{jk}^{0} \quad ; \; \varepsilon_{A} = \varepsilon_{A}^{L} + \sum_{j,k} D_{A,jk}\varepsilon_{jk}^{0}$$

a)



b)



c)

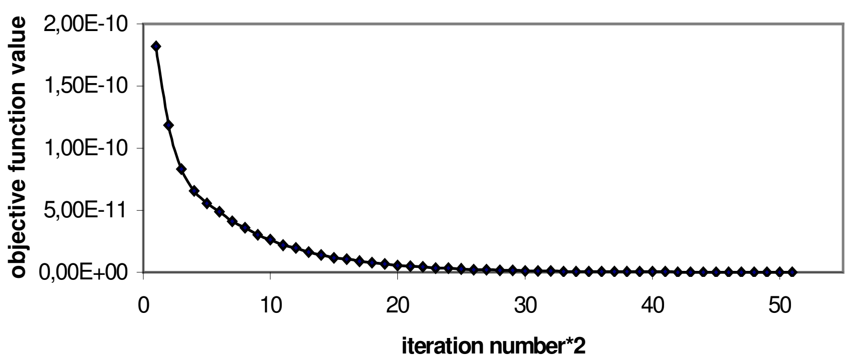


Fig.2(a) Cantilever truss structure consisting of 2 outer layers joined by the inner layer, which exhibits delamination in few locations (dashed lines). Sensors (elements able to detect strain) are placed in upper and lower horizontal members marked by bold lines. Static vertical force was applied at free end to identify the delamination defects.(b) virtual distortions and damage coefficient values obtained during optimization process. (c) optimization routine progress.

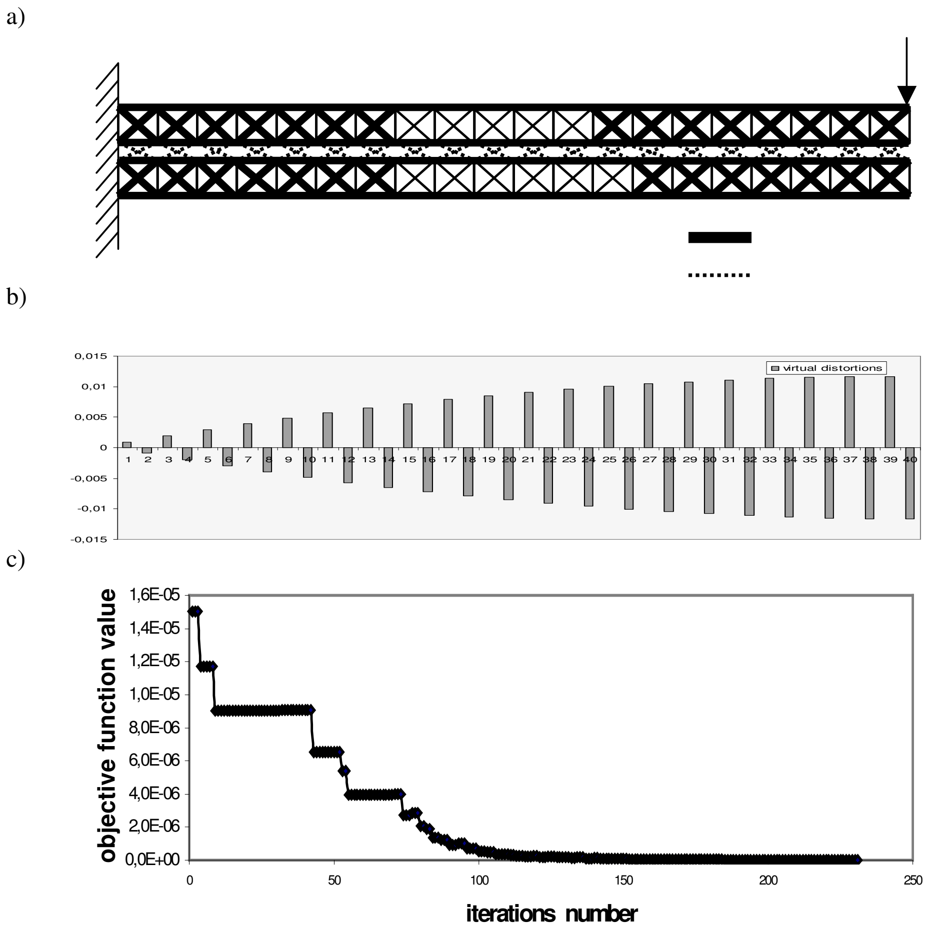


Fig.3. (a) Full delamination case. Sensors are marked by bold lines. Static vertical force was applied at free end to identify the delamination defects.(b) Damage coefficient values obtained in optimization process. (c) Optimization routine progress.

A simple truss model has been used demonstrating the problem of identification of delamination zone. The delaminated region has been modelled as a very thin layer (composed of truss elements also) placed between two thick layers

Numerical tests have been done for different delamination zone positions and sizes. Optimization routine was successful in finding defects, but large number of sensors was used, especially in the case of full delamination effect.

DYNAMIC CASE

The problem of identification of delaminations defined as a static problem leads to multi-sensor *observability*, which was demonstrated above. Let us demonstrate now, that the same problem, defined as a dynamic one, allows us to use only few sensors. Assuming impact excitation (generated by an actuator), transmitted along the beam and measured by a sensor located in a distance, the inverse dynamic analysis has to be performed in order to identify locations and intensities of defects. The Dynamic Virtual Distortion Method (Ref. 4), which is based on assumption that the virtual distortions depend on time, can be applied as the solver of the current identification problem. Both the structural response and influence matrix are time-dependent, and the formula for measured strain development (2) takes now the following form:

$$\varepsilon_A(t) = \varepsilon_A^L(t) + \varepsilon_A^R(t) = \varepsilon_A^L(t) + \sum_{\tau=0}^{t} \sum_{j,k} D_{A,jk}(t-\tau)\varepsilon_{jk}^0(\tau) \tag{8}$$

It is important to note, that time-dependent influence matrix is obtained for unit impulse excitations applied in time instant t=0 (it means that excitation has non-zero value only for one time step). The unit impulse excitation can be supplied in form of initial velocity conditions: $V(0) = \frac{P\Delta t}{m}$, where P denotes compensative force corresponding to locally generated unit virtual distortion impulse $\varepsilon^0 = 1$, Δt denotes the integration time step, and m - the mass concentrated in the loaded element. Having the influence matrix $D_{A,jk}(t)$ (in the case of only one sensor $D_{A,jk}(t)$ describes strain in observable element A, for each possible location of compensative forces) we can calculate the superposition of linear, time-dependent structural responses.

The inverse analysis leads to minimisation of the objective function describing differences between the measured response ε_A^M and the modelled one ε_A (expressed by Eq. 8). Finally, the defect identification problem takes the following form:

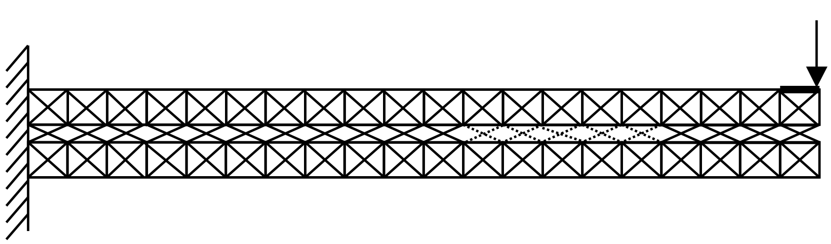
$$\min \sum_t (\varepsilon_A^M(t) - \varepsilon_A(t))^2 \tag{9}$$

subject to the following constraints, where μ is defined by the time-dependent version of the formula (3):

$$0 \le \mu_i \le 1 \tag{10}$$

240-element truss structure model has been used in the dynamic case presented below. Full sine impulse load has been applied in the tip point of the truss-beam cantilever structure (Fig. 3a) to generate the reference response as well as the response of damaged structure. In this case of load type, the influence of normal forces on structural response can be neglected, and the objective function can be expressed through variables ε_{jR}^0 and ε_{jL}^0 only. The VDM-modelled structure response has been compared vs. the case with shear-forces variation modelled by the corresponding variation of Young modulus in elements modelling contact layer.

a)



b)

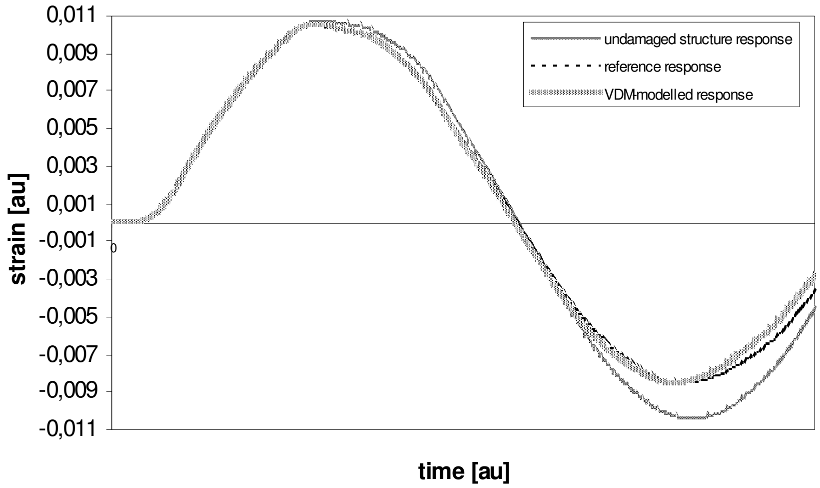


Fig.4 Dynamic VDM approach results. (a) The numerical model. (b) Damaged and undamaged structure responses.

5. CONCLUSIONS

Potential of use of the VDM-based approach to identification of delamination defect has been presented. It has been demonstrated that a relatively large number of sensors has to be applied in some cases in order to identify properly delamination defects in case of statically formulated problem. Then, it has been demonstrated (using the same, relatively simple numerical model) that the dynamic problem formulation is promising thanks to placing only few sensors for damage identification.

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REFERENCES

1. Rosen J. B. (1960) The Gradient Projection Method for Nonlinear Programming, Part I – Linear Constraints, *SIAM Journal of Applied Mathematics*, 8(1), pp. 181-217
2. Rosen J. B. (1961) The Gradient Projection Method for Nonlinear Programming, Part II – Nonlinear Constraints, *SIAM Journal of Applied Mathematics*, 9(4), pp. 514-532
3. Haug E. J., Arora J. S. (1979) *Applied Optimal Design: Mechanical and Structural Systems*, John Wiley & Sons, New York, U.S.A.
4. Holnicki-Szulc,J, T.Zielinski, *New damage Identification Method Through the Gradient Based Optimisation*, Proc. COST International Conference on System Identification & Structural Health Monitoring, Madrid, 6-9 June, 2000